Comparison of Interpolatory Methods for Parametric Model Order Reduction

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Interpolatory Methods for Parametric Model Order Reduction (PMOR)

Consider a parameterized dynamical system of order n and with corresponding transfer function

$$G(s,p) = C(p)(sE(p) - A(p))^{-1}B(p), \quad E(p), A(p) \in \mathbb{R}^{n\times n}, B(p) \in \mathbb{R}^{n\times m}, \ C(p) \in \mathbb{R}^{\ell\times n}.$$

Projection-based PMOR seeks for (full column rank) matrices $V_r . W_r \in \mathbb{C}^{n\times r}$ with $r \ll n$ such that the reduced-order transfer function

$$\hat{G}(s,p) = C(p)V_r(sW_r^T E(p)V_r - W_r^T A(p)V_r)^{-1}W_r^T B(p)$$

approximates the original well: $\hat{G}(s,p) \approx G(s,p)$.
We consider the following interpolatory methods for PMOR:

- (multi)parameter moment matching (MM) [1-4]

$$\frac{\partial^k}{\partial s^k}\frac{\partial^l}{\partial p^l}G(\tilde{s}_j,\tilde{p}_j) = \frac{\partial^k}{\partial s^k}\frac{\partial^l}{\partial p^l}\hat{G}(\tilde{s}_j,\tilde{p}_j), \quad j = 1,\dots,K, \ k = 0,\dots,q, \ l = 0,\dots,q,$$

- PMOR by matrix interpolation (MI) [5]

$$\hat{E}(p) = \sum_{j=1}^{K}\omega_j(p)M_jW_j^TE(p_j)V_jT_j^{-1}, \quad \hat{A}(p), \hat{B}(p), \hat{C}(p) = \dots, \quad M_j, \ T_j \in \mathbb{R}^{r\times r},$$

- interpolation of transfer functions (TFI) [6]

$$\hat{G}(s,p) = \sum_{j=1}^{K} L_j(p)C(p_j)V_j(sW_j^TE(p_j)V_j - W_j^TA(p_j)V_j)^{-1}W_j^TB(p_j),$$

- interpolation in tangent space to Grassmann manifold (IGM) [7-9]
- tangential interpolation (with $\mathcal{H}_2$ optimal frequency points) (TI) [10]

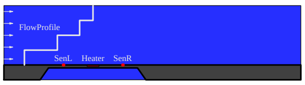
$$G(\tilde{s},\tilde{p})b = \hat{G}(\tilde{s},\tilde{p})b, \quad c^TG(\tilde{s},\tilde{p}) = c^T\hat{G}(\tilde{s},\tilde{p}), \quad \nabla_p c^TG(\tilde{s},\tilde{p})b = \nabla_p c^T\hat{G}(\tilde{s},\tilde{p})b.$$

Anemometer Example

Anemometer is a flow meter consisting of a heater and two temperature sensors before and after the heater in the direction of the flow.
After space discretization, we obtain

$$E\dot{x}(t) = (A_1 + vA_2)x(t) + Bu(t), \quad y(t) = Cx(t)$$

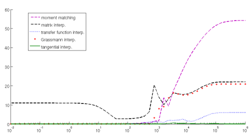
- $n = 29008$
- sparse matrices
- parameter: fluid velocity $v \in [0,1]$



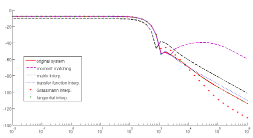
C. Moosmann, E.B. Rudnyi, J.G. Korvink, and A. Greiner, Model order reduction for linear convective thermal flow, THERMINIC 2004.

Results

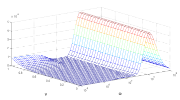
PMOR	r	max error	interp. points	nonparam. MOR	remarks	plot
MM	55	1.1e-01	$\tilde{p}_1 = 0$	$q = 10$	$\tilde{s}_1 = 0$	
	55	3.5e-03	$\tilde{p}_1 = 0.5$	$q = 10$	$\tilde{s}_1 = 100$	
	60	4.8e-03	4 Chebyshev	$q = 5$	4 $\tilde{s}_j \in [10, 1000]$	⋆
MI	60	1.4e-01	12 equidist.	Arnoldi	linear weights	⋆
	60	9.1e-01	12 Chebyshev	BT	Lagrange weights	
TFI	60	7.8e-04	12 Chebyshev	BT	Lagrange interp.	⋆
IGM	60	1.0e-02	12 Chebyshev	Arnoldi	Lagrange interp.	⋆
TI	60	1.7e-04	12 Chebyshev	IRKA		⋆



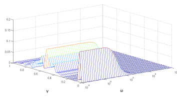
$max_{p\in[0,1]}|G(\omega,p) - \hat{G}(\omega,p)|$



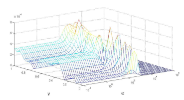
Frequency response for $p = 0.5$



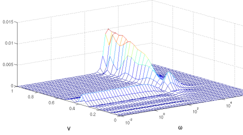
Absolute error with MM



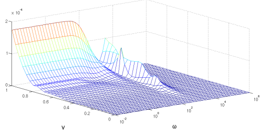
Absolute error with MI



Absolute error with TFI



Absolute error with IGM



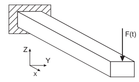
Absolute error with TI

Beam Example

We consider a FE model which describes the motion of a beam

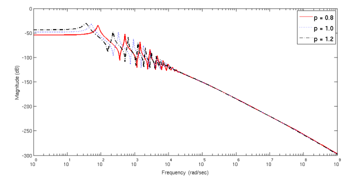
$$E(L)\dot{x}(t) = A(L)x(t) + Bu(t), \quad y(t) = Cx(t),$$

- $n = 1200$
- parameter: length of beam $L \in [0.8, 1.2]$
- non-affine parameter-dependency in E and A
- peaks



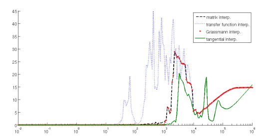
H. Panzer, J. Hubele, R. Eid, B. Lohmann, Generating a parametric finite element model of a 3d cantilever timoshenko beam using matlab, techn. report, Technische Universität München, 2009.

Frequency response for $L \in \{0.8, 1.0, 1.2\}$:

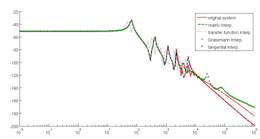


Results

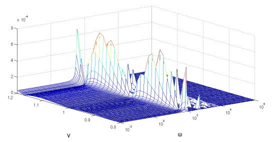
method	r	max error	interp. points	nonparam. MOR	remarks	plot
MI	30	2.2e-03	$\{0.8, 1.0, 1.2\}$	Arnoldi	hat function	
	30	7.0e-04	3 Chebyshev	Arnoldi	Lagrange weights	⋆
TFI	30	2.8e-02	3 Chebyshev	BT	Lagrange interpol.	
	30	2.0e-02	3 Chebyshev	Arnoldi	Lagrange interpol.	⋆
IGM	30	2.3e-04	3 Chebyshev	Arnoldi	Lagrange interp.	⋆
TI	30	2.6e-05	3 Chebyshev	IRKA		⋆



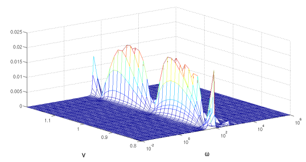
$max_{p\in[0.8,1.2]}|G(\omega,p) - \hat{G}(\omega,p)|$



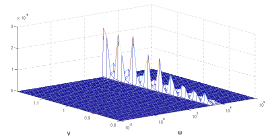
Frequency response for $p = 0.9$



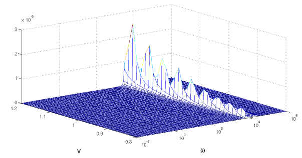
Absolute error with MI



Absolute error with TFI



Absolute error with IGM



Absolute error with TI

Literature

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[8] D. Amsallem, Interpolation on manifolds of CFD-based fluid and finite element-based structural reduced-order models for on-line aerolastic predictions, dissertation, department of aeronautics and astronautics, Stanford University, 2010.

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